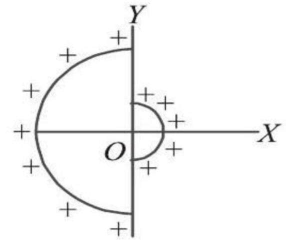
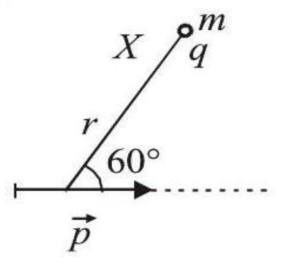
Electric Charges and Fields

1. Two semicircular wires of radius 20 cm and 10 cm have a common centre at the origin O as shown in the figure. Assume that both the wires are uniformly charged and have an equal charge of 0.70 nC each. The magnitude of electric field (in Vm^{-1}) at the common centre of curvature O of the system is



2. Three point charges Q_1, Q_2, Q_3 in the order are placed equally spaced along a straight line. Q_2 and Q_3 are equal in magnitude but opposite in sign. If the net force on Q_3 is zero, the value of Q_1 is $x(Q_3)$. Find the value of x.
3. A uniformly charged conducting sphere of 4.4 m diameter has a surface charge density of 60μ m m^{-2}. The charge (in coulomb) in coulamb on the sphere is
4. When an electric dipole $\vec{P}$ is placed in a uniform electric field $\vec{E}$ then at what angle between $\vec{P}$ and $\vec{E}$ the value of torque will be maximum?
5. An uniform electric field E exists along positive x -axis. The work done in moving a charge 0.5 C through a distance 2 m along a direction making an angle 60° with x -axis is 10 J. Then the magnitude of electric field (in Vm^{-1}) is
6. Two point charges $+3\mu C$ and $+8\mu C$ repel each other with a force of $40N$. If a charge of $-5\mu C$ is added to each of them, then the force (in newton) between them will become
7. Two equally charged identical metal spheres A and B repel each other with a force 2.0×10^{-5} N. Another identical uncharged sphere C is touched to A and then placed at the mid point between A and B. What is the net electric force (in newton) on C ?
8. A sphere of radius R carries charge such that its volume charge density is proportional to the square of the distance from the center. What is the ratio of the magnitude of the electric field at a distance 2R from the center to the magnitude of the electric field at a distance of $\mathrm{R}/2$ from the center?
9. A negatively charged oil drop is prevented from falling under gravity by applying a vertical electric field of $100\mathrm{Vm}^{-1}$. If the mass of the drop is 1.6×10^{-3} g, the number of electrons carried by the drop is ($\mathrm{g} = 10\ \mathrm{ms}^{-2}$)
10. An electric dipole is placed at an angle of 30° with an electric field of intensity $2 \times 10^{5}\mathrm{NC}^{-1}$, It experiences a torque of 4 Nm. Calculate the charge (in mC) on the dipole if the dipole length is 2 cm.
11. A point charge of charge q and mass m is placed at rest at point X at distance r from a short electric dipole. The initial acceleration of charge $a = \frac{k \cdot q \cdot p}{2mr^3} n$, where $k = \frac{1}{4\pi\varepsilon_0}$. Then n is

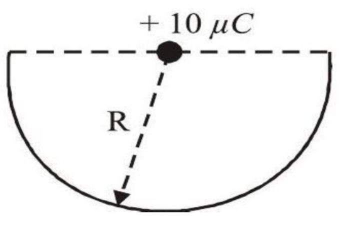


(short electric dipole)

12. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = \kappa r^a$, where κ and a are constants and r is the distance from its centre.

If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at $r = R$, find the value of a.

13. A charge of $10 \mu C$ is placed at the centre of a hemisphere of radius $R = 10$ cm as shown. The electric flux through the hemisphere (in MKS units) is

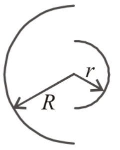


14. A point charge causes an electric flux of $-1.0 \times 10^3 Nm^2/C$ to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge of the radius of the Gaussian surface were three times, how much flux would pass through the surface.

15. An electric field of 1000 V/m is applied to an electric dipole at an angle of $45°$. The value of electric dipole moment is 10^{-29}C. m. What is the potential energy (in joule) of the electric dipole?

SOLUTIONS

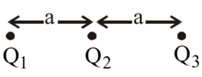
1. **(300.96)** $E_{Net} = 2K\left[\frac{Q}{\pi r^2} - \frac{Q}{\pi R^2}\right]$



$= \frac{2KQ}{\pi}\left[\frac{1}{r^2} - \frac{1}{R^2}\right]$

$= \frac{2 \times 9 \times 0.7 \times 10^9 \times 10^9}{3.14 \times 7.5} = 300.96$ V/m

2. **(4)**



; $Q_2 = -Q_3 = Q$

Force on Q_3 due to Q_2 + Force on Q_3 due to $Q_1 = 0$.

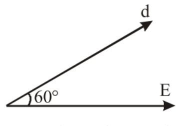
$\frac{1}{4\pi \epsilon_0}\left(\frac{-Q^2}{a^2}\right) + \frac{1}{4\pi \epsilon_0}\frac{Q_1 Q}{4a^2} = 0 \Rightarrow Q_1 = 4Q_3$

3. **(3.7×10^{-3})** Here, $D = 2r = 4.4\ m$,
or $= r = 2.2\ m; \sigma = 60\ \mu C\ m^{-2}$
Charge on the sphere, $q = \sigma \times 4\pi r^2$

$= 60 \times 10^{-6} \times 4 \times \frac{22}{7} \times (2.2)^2 \simeq 3.7 \times 10^{-3}$ C

4. **(90°)** When $\theta = 90°$, torque will have maximum value and this maximum value will be,
$\tau_{max} = PE \sin 90°$
$\tau_{max} = PE$

5. **(20)** Force acting on the charged particle due to Electric field $= q\vec{E}$



work done in moving through distance S,

$W = q\vec{E}.\vec{d} = (qE) \times d \times \cos\theta$

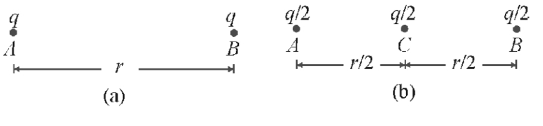
$\therefore\ 10\,J = (0.5\,C) \times E \times 2\cos 60°$

$E = 10 \times 2 = 20\ Vm^{-1}$

6. **(10)**

7. **(2×10^{-5})** Suppose charge of each sphere is q, the force between spheres A and B for separation r,

$F = 2.0 \times 10^{-5} = \frac{1}{4\pi \epsilon_0}\frac{q^2}{r^2}$ ...(i)



When uncharged sphere C is touched to the sphere A, the charge on each one becomes $q/2$. Thus force between A and C

$$F_{AC} = \frac{1}{4\pi \in_0} \frac{(q/2)(q/2)}{(r/2)^2} = F,$$

Force between B and C,

$$F_{BC} = \frac{1}{4\pi \in_0} \frac{\left(\frac{q}{2}\right) q}{\left(\frac{r}{2}\right)^2} = 2F.$$

Thus the net force on the sphere C

$= 2F - F = F = 2.0 \times 10^{-5}$ N from C to A

8. **(2)** $\rho = Cr^2$

$$q = \int_0^r 4\pi r^2 dr\rho = \int_0^r 4\pi C r^4 dr = \frac{4}{5}\pi C r^5$$

$$E_{r=2R} = \frac{kq_{(r=R)}}{(2R)^2} = \frac{k(4/5)\pi C R^5}{4R^2}$$

$$E_r = \frac{R}{2} = \frac{kq_{(r=R)}}{(R/2)^2} = \frac{k(4/5)\pi C(R/2)^5}{(R/2)^2}$$

Now solve to get $\frac{E_{r=2R}}{E_{r=R/2}} = 2$

9. **(10^{12})** For the drop to be stationary, $qE = mg$

$$\therefore q = \frac{1.6 \times 10^{-6} \times 10}{100} = 1.6 \times 10^{-7} C$$

Number of electrons in drop

$$= \frac{q}{e} = \frac{1.6 \times 10^{-7} C}{1.6 \times 10^{-19} C} = 10^{12} \text{ electrons}$$

10. **(2)** Torque, $\vec{\tau} = \vec{p} \times \vec{E} = pE \sin\theta$

$4 = p \times 2 \times 10^5 \times \sin 30°$

$$\text{or, } p = \frac{4}{2 \times 10^5 \times \sin 30°} = 4 \times 10^{-5} \text{Cm}$$

Dipole moment, $p = q \times l$

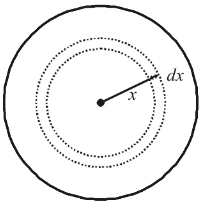
$$q = \frac{p}{l} = \frac{4 \times 10^{-5}}{0.02} = 2 \times 10^{-3} C = 2mC$$

11. **(2.6)** $$a = \frac{F}{m} = \frac{Eq}{m} = \left[\frac{1}{4\pi\varepsilon_0} \frac{P\sqrt{3\cos^2 60° + 1}}{r^3}\right] \frac{q}{m}$$

$$= \frac{q}{m}\left[\frac{1}{4\pi\varepsilon_0} \frac{P\sqrt{3\left(\frac{1}{2}\right)^2 + 1}}{r^3}\right]$$

$$= \frac{\sqrt{7}}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{qP}{mr^3}.$$

12. **(2)** Let us consider a spherical shell of radius x and thickness dx. The volume of this shell is $4\pi x^2(dx)$. The charge enclosed in this spherical shell is



$dq = (4\pi x^2)dx \times kx^a$

$\therefore \quad dq = 4\pi k x^{2+a}\, dx.$

For r = R :

The total charge enclosed in the sphere of radius R is

$$Q = \int_0^R 4\pi k\, x^{2+a}\, dx = 4\pi k \frac{R^{3+a}}{(3+a)}.$$

$\therefore$ The electric field at $r = R$ is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k R^{3+a}}{(3+a)R^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{(3+a)}R^{1+a}$$

For r = R/2 :

The total charge enclosed in the sphere of radius $R/2$ is

$$Q' = \int_0^{R/2} 4\pi k\, x^{2+a} dx = \frac{4\pi k (R/2)^{3+a}}{(3+a)}$$

$\therefore$ The electric field at $r = R/2$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}\frac{(R/2)^{3+a}}{(R/2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{(3+a)}\left(\frac{R}{2}\right)^{1+a}$$

Given, $E_2 = \frac{1}{8}E_1$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{(3+a)}\left(\frac{R}{2}\right)^{1+a} = \frac{1}{2^3}\times\frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{(3+a)}R^{1+a}$$

$\Rightarrow \quad 1 + a = 3 \Rightarrow a = 2$

13. **(6×10^5)**

14. **(1×10^3)** According to Gauss's theorem electric flux through a closed surface S = q/ε_0

Where q = total charge enclosed by S

Electric flux pass through the spherical of Gaussian surface is independent on the radius of a Gaussian surface but depends on the charge enclosed by a Gaussian surface. On increasing the radius of Gaussian surface three times, but the charge enclosed by a Gaussian surface remain the same. Therefore, the electric flux pass through the Gaussian surface remains the same.

15. **(-7×10^{-27})** Potential energy of a dipole is given by

$U = -\vec{P}.\vec{E}$

$= -PE\cos\theta$

[Where θ = angle between dipole and perpendicular to the field]

$= -(10^{-29})(10^3)\cos 45°$

$= -0.707 \times 10^{-26}$ J

$= -7 \times 10^{-27}$J